Modeling flow in the vertical plane with GFLOW

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Introduction

The GFLOW modeling system is designed primarily for modeling regional groundwater flow in the horizontal plane, adopting the Dupuit-Forchheimer approximation. However, the program can be used for modeling flow in a vertical cross-section over one or more aquifers, provided that there is no flow perpendicular to the plane of the cross-section and provided that the flow system is confined. The purpose of this document is to provide guidance in setting up such cross-sectional models.

General concepts

The idea of using a regional flow model (horizontal flow model) for modeling flow in the vertical plane is to imagine an aquifer of unit thickness (1 foot or 1 meter) and turn it upright. Hence, the aquifer thickness of 1 foot or 1 meter represents a vertical slice of the actual aquifer or aquifer system. The aquifer bottom and aquifer top in the GFLOW model form two vertical no-flow surfaces (planes) between which the groundwater flows. The bottom and top of the actual aquifer or aquifers must be introduced by use of no-flow boundaries ("horizontal barriers"). Head specified boundaries are introduced by line-sinks. These line-sinks may have resistance to flow, for instance modeling the interaction between the aquifer and an overlying wetland or lake with bottom resistance. The line-sink width should be set to unity (1 meter or 1 foot) representing the width of the cross sectional model. A well in the GFLOW model domain would have a screen over the 1 foot or 1 meter thick aquifer slice and thus represents a (infinitely long) horizontal drain in the aquifer. Wells *cannot* be represented in the aquifer, since they would generate flow perpendicular to the vertical plane inside the 1 meter or 1 foot thick aquifer slice. To ensure saturated flow inside the vertical aquifer slice under all conditions, the head must be larger (or at least equal) to the aquifer top in GFLOW (aquifer base + aquifer height on the *Model>Settings>Aquifer* tab) everywhere in the model domain. This can be accomplished by setting the aquifer bottom elevation at least 1 foot below the lowest specified head. In case of discharge specified features the aquifer bottom

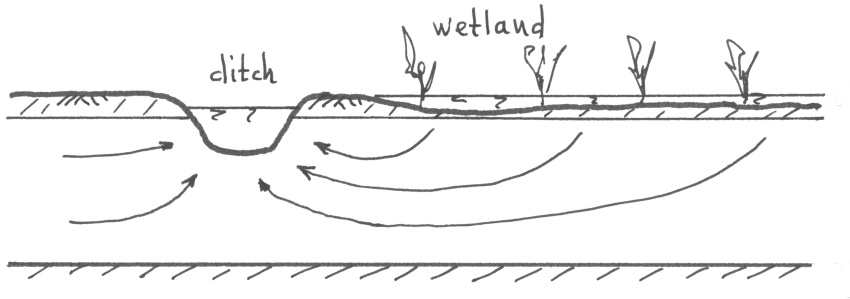

Figure 1: A cross-section over an aquifer with a ditch, a wetland and a horizontal drain.

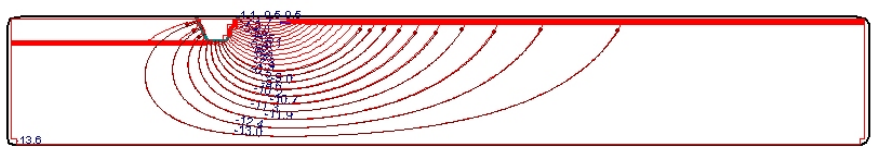

Figure 2: GFLOW cross-sectional model of the case in Figure 1 with some streamlines. Project file: *example1.gfl*

elevation must be at least 1 foot below the *anticipated* lowest head (probably near the discharge specified feature).

Example

As an example of a cross-sectional flow model consider the situation depicted in Figure 1. It exhibits a ditch to the left of a wetland. The hydraulic conductivity of the aquifer is $k = 10\ m/day$ and the height of the aquifer is about 20 meters. The water level in the ditch is equal to the head in the aquifer at the ditch sides and bottom and is 20 meters. The water level in the wetland is 22 meters and the wetland bottom is about 1 meter thick with a vertical hydraulic conductivity of 1 m/day.

A cross-sectional GFLOW model has been designed to represent the flow system in Figure 1. A picture of the element layout and some streamlines are shown in Figure 2. The aquifer is surrounded by a no-flow boundary, which has been created using the "horizontal barrier" feature in GFLOW, selecting a "closed domain" and selecting "ignore outside region", hence no heads or discharges will be calculated outside the closed domain. Note that the corners of the domain have been rounded to improve the solution to the no-flow boundary. Not visible in Figure 2 is the fact that near the corners and near the ditch the vertex density of the boundary has been increased. This may be viewed by

opening the project file "example1.gfl" and clicking on the boundary to highlight it and show the vertices.

The ditch is represented by three line-sink strings that are marked "treat as farfield," hence have no resistance, width or depth parameters set. The specified head is the head in the aquifer at the line-sink centers and is set to 20 meters.

The wetland has been entered as a string of line-sinks with a resistance of 1 day (1 meter thick layer divided by 1 m/day conductivity). The width of the line-sink is set to 1 meter, which is consistent with the fact that the line-sinks occurs in a 1 meter wide aquifer slice. This setting is important to arrive at the proper implementation of the resistance to flow through the top 1 meter of low permeable soil. The depth parameter of the line-sink string is set to 2 meters, which is the distance between the surface water elevation in the wetland and the bottom of the low permeable soil below the wetland. It should be recalled that, when the head in the aquifer drops more than the depth parameter below the water level in the wetland, the wetland is assumed percolating. We cannot accept this condition, since the flow in the cross-sectional model must be confined. Consequently, if the GFLOW solution shows percolating wetland line-sinks, the cross-sectional model becomes inaccurate. You make check for this selecting "highlight percolating linesinks" on *View>Results Overlay...*

Managing branch cuts

There are two sets of streamlines shown in Figure 2: the light red ones are contours of the **stream function** and the dark red ones are path line traces from particles that have been positioned on some of these streamlines. The display of these stream lines and path lines requires some explanation. First of all, there is a thick red line extending from the bottom of the ditch to the left-hand boundary of the model domain; this is a branch cut for the line-sinks along the bottom of the ditch. The behavior of the stream function is discussed in Haitjema (1995) pages 50-51 and shown for the case of a well. Each feature that adds or removes water from an aquifer has a branch cut in the stream function associated with it. Since in Figure 2 the line-sinks along the bottom of the ditch start and end inside the domain, the branch cut must also be inside the domain. The line-sinks for the wetland are positioned along the boundary of the domain and the associated branch cut is also along the boundary. The line-sinks along the sides of the ditch have been entered starting at the domain boundary and ending inside the domain. As a result, the branch cuts for these two line-sink strings run along the sides of the ditch and extend upward away from the model domain. Since the domain outside the no-flow boundary has been flagged to be ignored, these branch cuts are not visible above the model domain.

The branch cut is formed by bundled-up contours between two grid lines of the contour grid where the stream function jumps. This jump in stream function value equals the extraction rate of the line-sink string. As a result, a stream line that crosses the branch cut does not have the same stream function value. Consequently, the stream line stops at the branch cut and other streamlines

are continued at the other side. This is clearly visible in the upper left part of the model domain in Figure 2. Similarly, the line-sink string that represents the wetland has been started at the right-hand aquifer boundary so that its branch cut extends to the right outside the model domain, rather than to the left through the upper part of the aquifer and ditch.

To illustrate that the contouring of the stream function indeed yields streamlines, particles have been placed on some of these contours and path line tracing has been activated on the *Model>Settings>Tracing* tab. The path lines (dark red lines) closely follow the streamlines (light red lines), except above the branch cut to the left of the ditch. However, those streamlines are not continuations of the streamlines (stream function contours) below the branch cut, as discussed.

Use of the stream function

Contours of the stream function only represent streamlines for groundwater flow problems governed by the equation of Laplace. This is rarely the case for regional flow problems, where areal recharge due to precipitation or leakage from adjacent aquifers gives rise to groundwater flow problems governed by Poisson's equation. However, cross-sectional models typically do satisfy Laplace's equation since no flow is allowed perpendicular to the vertical plane, hence no areal sinks or sources exist in the plane of the cross-section. It is important to realize that if, for some reason, a recharge inhomogeneity is included in the model domain, stream function contours still exist, but they do not represent streamlines!

Dealing with remote boundary conditions

The lateral extent of aquifers is usually orders of magnitude larger than the aquifer thickness. This circumstance means that flow is mostly horizontal, except near a feature such as a shallow ditch, see Figure 1 and Figure 2. The left-hand and right-hand (vertical) aquifer boundaries in Figure 2 are artificial and do not represent the actual flow system where the wetland to the right of the ditch may extend for hundreds of meters or several kilometers and where the aquifer to the left of the ditch may also extend very far before a boundary condition is encountered, say a Dirichlet boundary (head specified boundary). Explicitly representing a model domain that is kilometers long (and say 20 meters in height) is often impractical, if not impossible. Such a model domain would become very long and thin, requiring many analytic elements to represent the aquifer boundary and the wetlands. The flow in the aquifer, away from the ditch, would also become rather uninteresting: almost exclusively horizontal flow. Incidentally, such very elongated domains are particularly problematic in grid-based models, such as MODFLOW. The long thin model could pose serious grid resolution problems.

It is possible to include the remote aquifer sections and remote boundary conditions in a local cross-sectional model without explicitly representing the

aquifer in those areas. This is done in Figure 3 and Figure 4 by adding a line-sink string along both the vertical left-hand and right-hand aquifer boundary.

Including a remote Dirichlet boundary

The line-sink string along the left-hand boundary has been given a resistance to represent the resistance to flow in the aquifer to the left of the model domain. It is assumed that there is a head-specified boundary at a distance of L meters to the left of the model domain. The resistance to flow follows from Darcy's law as follows:

$$q_x = k\frac{\phi_1 - \phi_l}{L} = \frac{\phi_1 - \phi_l}{c} \tag{1}$$

where ϕ_1 is the head at a distance L from the left-hand model domain boundary, and where ϕ_l is the head at the left-hand *model* domain boundary shown in Figure 3 and Figure 4. The resistance c follows from (1) as:

$$c = \frac{L}{k} \tag{2}$$

The line-sink string along the vertical left-hand aquifer boundary, therefore, forms a Cauchy type boundary condition or a "head-dependent flux boundary". This approach to including a remote Dirichlet boundary in a local model is routinely used by MODFLOW modelers employing a "general head boundary."

Including a remote wetland boundary

The line-sink string along the right-hand boundary has also been given a resistance to represent the resistance to flow from the wetland into the aquifer and through the aquifer that occurs to the right of the model domain boundary. This resistance is determined as follows. We will assume that the wetland extends infinitely far to the right of the model domain. For that case, the total flow out of the wetland that enters the model domain through the right-hand boundary is given by Verruijt (1970), page 30 equation (4.10), as:

$$Q = kHB\frac{\phi_w - \phi_r}{\lambda} \tag{3}$$

where ϕ_w is the water level (head) in the wetland and ϕ_r the head in the aquifer at the right-hand model boundary. The parameter H is the aquifer thickness and B is the aquifer width, which is the width of the 1 meter slice for our case. The parameter λ is referred to here as the *characteristic leakage length* and defined as (Verruijt, 1970):

$$\lambda = \sqrt{kHc_w} \tag{4}$$

where c_w is the resistance of the wetland bottom, which is the thickness of the wetland bottom divided by its vertical hydraulic conductivity. Equation (3), when divided by H and setting $B = 1\ m$, represents the specific discharge at the right-hand model domain boundary:

$$q_x = -k\frac{\phi_w - \phi_r}{\lambda} \tag{5}$$

which can be compared to flow in a fictitious confined aquifer with Dirichlet boundary conditions:

$$q_x = -\frac{\phi_w - \phi_r}{c} \tag{6}$$

The minus signs in (5) and (6) reflect the fact that the flow is from the right to the left, thus in negative x-direction. Comparing (5) with (6) yields a resistance c for the line-sink string on the right-hand domain boundary of:

$$c = \frac{\lambda}{k} \tag{7}$$

In defining these surrogate boundaries the heads along the line-sinks are set equal to the known heads ϕ_w and ϕ_1, respectively. The heads ϕ_l and ϕ_r, that occur along the left-hand and right-hand domain boundaries, do not have to be known. Also, in defining the resistance for the right-hand line-sink string it does not matter where the model domain boundary is. The equations (1) through (7) have been written assuming Dupuit-Forchheimer flow to the left and to the right of the model domain in Figure 3 and Figure 4. To assure this, the left-hand and right-hand domain boundaries must be at least 2 times the aquifer thickness H away from the shallow ditch.

Example

The following data have been used in Figure 3 and Figure 4: $k = 100\ m/day$, $H = 20\ m$, $\phi_1 = 22\ m$, $\phi_w = 22\ m$, the head at the ditch is 20 m, $L = 500\ m$ (distance from left-hand boundary of the model domain to the remote Dirichlet condition), and the resistance of the wetland is $c_w = 1\ days$. In this example the left-hand and right-hand domain boundaries occur at a distance of 45 meters from the center line of the ditch, which is set at x=0. Consequently, the remote Dirichlet boundary occurs at a distance of 545 meters from the center line of the ditch, which is at x=-545, see Figure 3. The bottom of the ditch has been made a no-flow boundary, hence there will be no branch cut protruding into the model domain. The characteristic leakage length λ follows from (4) as:

$$\lambda = \sqrt{kHc_w} = \sqrt{100 * 20 * 1} = 44.7214\ m \tag{8}$$

The resistance for the left-hand line-sink string follows from (2) as:

$$c = \frac{L}{k} = \frac{500}{100} = 5\ days \tag{9}$$

The resistance for the right-hand domain boundary follows from (7) as:

$$c = \frac{\lambda}{k} = \frac{44.7214}{100} = 0.447214\ days \tag{10}$$

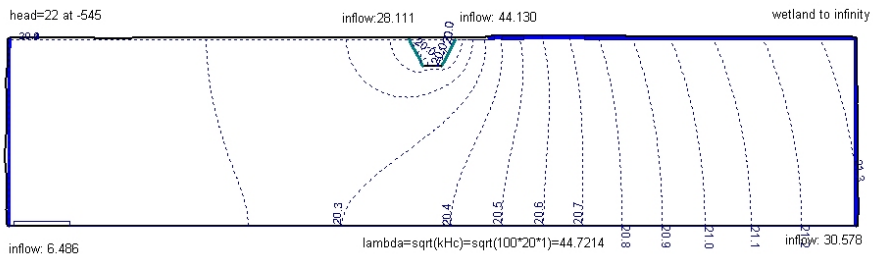

Figure 3: Potentiometric head contours near ditch. Model domain truncated on the left- and right-hand side with line-sink strings representing flow from remote aquifer zones. Project file: *example2.gfl*

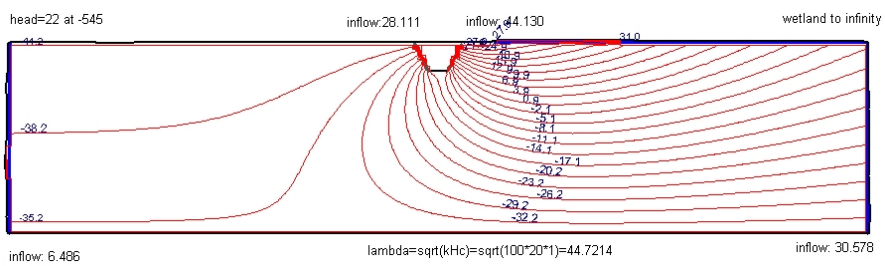

Figure 4: Streamlines (stream function contours) near ditch. Model domain truncated on the left- and right-hand side with line-sink strings representing flow from remote aquifer zones. Project file: *example2.gfl*

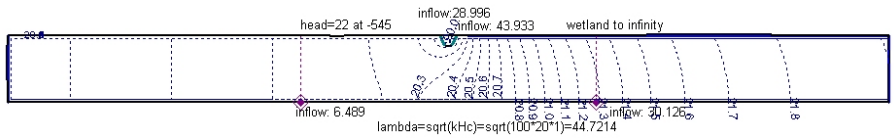

Figure 5: Potentiometric head contours in extended model domain. Model domain truncated on the left- and right-hand side with vertical line-sink strings representing flow from remote aquifer zones. Project file: *example2long.gfl*

The line-sink strings along the right- and left-hand boundaries occur at 45 meters from the center of the ditch, which is approximately two times the aquifer thickness ($H = 20\ m$) from the ditch. This will ensure that the Dupuit-Forchheimer approximation may be adopted near those boundaries and beyond.

Moving the boundaries further away from the ditch

To verify the validity of the approximations presented above, the same flow problem is solved again, but this time the boundaries are set at 134 meters from the center of the ditch, which is about equal to 3λ. It can be shown that over this distance the wetlands have infiltrated 95% of their total infiltration (Hunt et al. 2003). The resistance of the left-hand line-sink string must be recalculated using (2) as:

$$c = \frac{L}{k} = \frac{411}{100} = 4.11\ days \tag{11}$$

since the distance between the left-hand boundary and the Dirichlet boundary condition is now 545 -134=411 meters. The resistance for the right-hand line-sink string is still given by (10), thus does not depend on the location of this line-sink string!.

In Figure 5 and Figure 6 contour plots of the heads and streamlines are shown for this new configuration. At 45 meters from the center line of the ditch (thus at the model boundaries in the previous example, see Figure 3), the total discharge is measured using GFLOW's "flux inspector" feature. Comparing those flows to the ones obtained for the shorter model demonstrates that the inflows are nearly the same for both boundary configurations. In other words, the truncated model in Figure 3 and Figure 4 provides the same flow patterns as the model with the boundaries moved nearly three times as far away from the ditch. The inflow to the left for the short and the long model is 6.486 m^2/day and 6.489 m^2/day respectively. The difference is 0.05%. The inflow to the right for the short and the long model is 30.578 m^2/day and 30.126 m^2/day respectively. The difference is 1.5%.

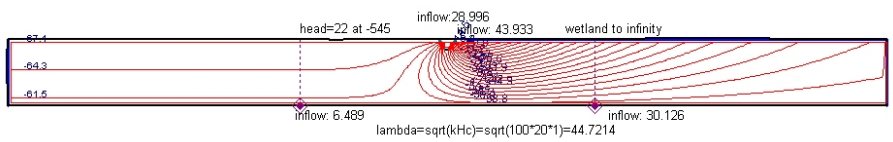

Figure 6: Streamlines (stream function contours) in extended model domain. Model domain truncated on the left- and right-hand side with vertical line-sink strings representing flow from remote aquifer zones. Project file: *example2long.gfl*

Refinement of resistance on boundary line-sinks

The left-hand boundary in the truncated model produces just about the same inflow as is found in the elongated model. The difference in the right-hand inflow at 45 meters from the ditch is 1.5%, however. While small, it is two orders of magnitude larger than for the left-hand boundary. The reason for this discrepancy is found in adoption of the Dupuit-Forchheimer assumption, by which we ignored resistance to vertical flow in the aquifer itself. In both the truncated and elongated model we used 10 line-sinks of 2 meters length to represent the left- and right-hand boundaries. Each line-sink was given the same resistance. This is correct for the line-sinks on the left-hand boundary, where the resistance to flow for each line-sink is the same: the distance between the remote Dirichlet condition and the model boundary divided by the aquifer hydraulic conductivity. This resistance is indeed the same for each line-sink. This is different for the right-hand boundary. Here we obtained the resistance of flow by ignoring resistance to vertical flow, but that is an approximation. As may be seen from Figure 4 and Figure 6 there is *some* vertical flow and thus also *some* resistance to vertical flow. This explains the 1.5% difference in the short and long model.

Resistance to vertical flow inside the aquifer can be included in an approximate manner by simply adding it for each individual line-sink as follows. The line-sink with its center at a distance d below the aquifer top (wetland bottom) is given a resistance as if the upper d meters of the aquifer is part of the wetland bottom. Consequently, the "effective resistance" of the wetland bottom c_d for that line-sink is calculated as:

$$c_d = c_w + \frac{d}{k} \tag{12}$$

where c_w is the actual resistance of the wetland bottom and k the hydraulic conductivity of the aquifer. Similarly a new characteristic leakage length λ_d is calculated for the line-sink with its center at depth d, as:

$$\lambda_d = \sqrt{kHc_d} \tag{13}$$

Finally, the resistance for the line-sink with its center at depth d is calculated, see (7), as:

$$c = \frac{\lambda_d}{k} \tag{14}$$

The use of (12) through (14) leads to a resistance along the line-sinks on the right-hand boundary that increases with depth. Implemented in the above presented case yields the streamline contours depicted in Figure 7 and Figure 8, whereby the last figure shows a zoom in on approximately the same domain as depicted in Figure 7. The total discharge through the aquifer at 45 meters from the center of the ditch is now 29.854 m^2/day and 30.002 m^2/day for the short and long model, respectively, which is a difference of about 0.5%.

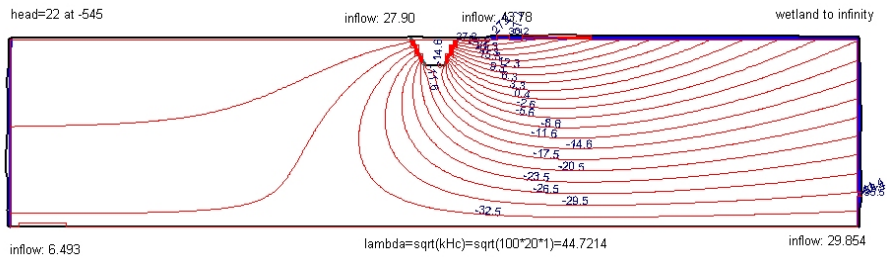

Figure 7: Streamlines (stream function contours) near ditch. Line-sinks along the right-hand boundary have a varying resistance. Project file: *example2refined.gfl*

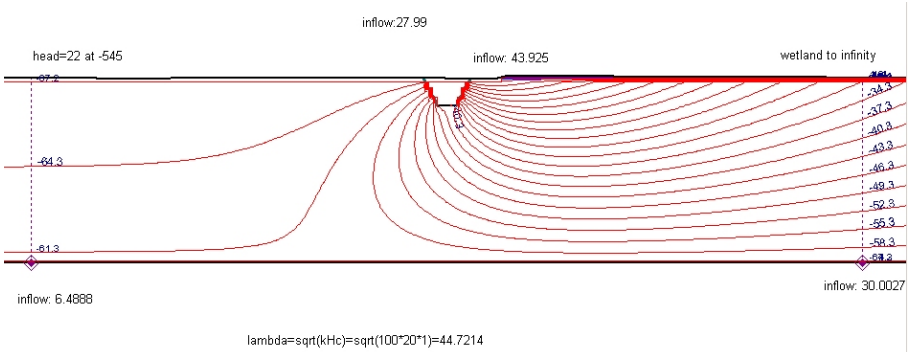

Figure 8: Streamlines (stream function contours) in extended model domain, but zoomed in to about the same domain as shown in Figure 7. Line-sinks along the right-hand boundary have a varying resistance. Project file: *example2longrefined.gfl*

Stratified aquifers

Cross-sectional models become more interesting when aquifer heterogeneities are included, such as aquifer stratification. In Figure 9 and Figure 10 the same flow system is shown as in the previous example, except there is a 10 meter thick layer in the center of the aquifer with a lower hydraulic conductivity: $k = 20$ m/day. Figure 10 is a zoom in of a long model, similar to the domain shown in Figure 6.

The left-hand and right-hand line-sink boundaries now have resistances that are specific to the aquifer layer in which the line-sinks occur. In calculating the resistances for the line-sinks on the left-hand boundary, for the short model domain, the aquifer strata are seen as independent aquifers, each offering its own resistance to (horizontal) flow. For the line-sinks in the upper and lower (5-meter thick) layer the resistance is still given by (9), hence $c = 5$ $days$. The resistance for the line-sinks in the center layer becomes:

$$c = \frac{L}{k} = \frac{500}{20} = 25 \ days \tag{15}$$

For the extended model domain the line-sink resistance for the upper and lower layer are still defined by (11), hence $c = 4.11$ m/day. For the center layer we find:

$$c = \frac{L}{k} = \frac{411}{20} = 20.55 \ days \tag{16}$$

The resistances for the line-sinks on the right-hand side require the recalculation of the characteristic leakage length λ, as follows, see also (4):

$$\lambda = \sqrt{(\sum(kH))c_w} \tag{17}$$

where the sum of kH is the total transmissivity in the aquifer, hence with the data for this case:

$$\lambda = \sqrt{(\sum(kH))c_w} = \sqrt{(100 * 5 + 20 * 10 + 100 * 5) * 1} = 34.641 \ meters \tag{18}$$

The resistances for the line-sinks in the various aquifer strata now follows from (7) for the two layers with $k = 100$ m/day as:

$$c = \frac{\lambda}{k} = \frac{34.641}{100} = 0.34.641 \tag{19}$$

and for the center layer with $k = 20$ m/day as:

$$c = \frac{\lambda}{k} = \frac{34.641}{20} = 1.7321 \tag{20}$$

These resistances apply to both the short and the long model domain.

The flow rates in the aquifer at 45 meters from the center of the ditch (location of the line-sink boundary in the short model) are nearly the same on

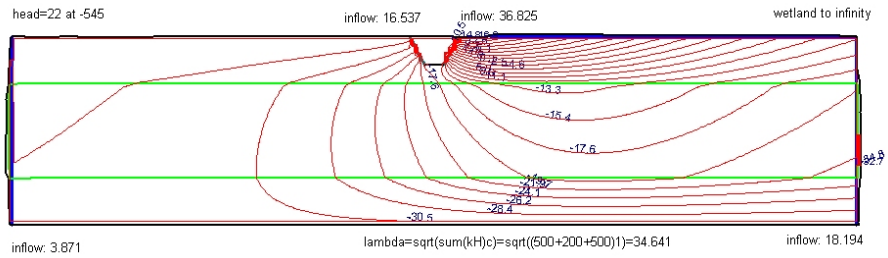

Figure 9: Streamlines (stream function contours) near ditch. Line-sinks along the right-hand and left-hand boundaries have different resistances depending on the aquifer layer they are in. Center layer has 5 times lower conductivity than adjacent layers. Project file: *example3.gfl*

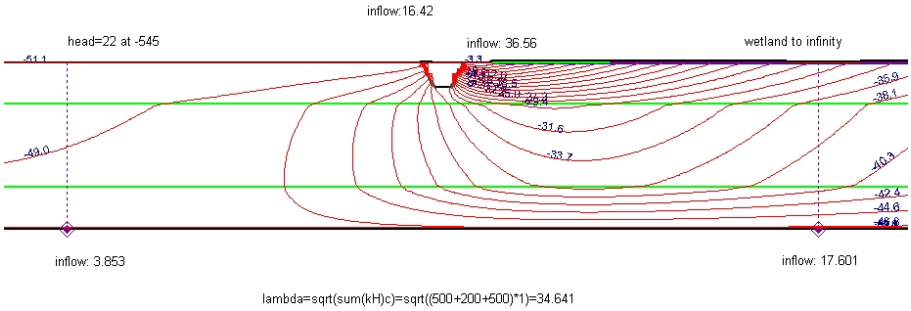

Figure 10: Streamlines (stream function contours) in extended model domain, but zoomed in to about the same domain as shown in Figure 7. Line-sinks along the right-hand and left-hand boundaries have different resistances depending on the aquifer layer they are in. Center layer has 5 times lower conductivity than adjacent layers. Project file: *example3long.gfl*

the left, but differ by about 3% on the right. This small discrepancy in the flow on the right-hand side is has the same cause as the 1.5% discrepancy found in the homogeneous case, see Figure 4 and Figure 6. In this case, however, ignoring resistance to vertical flow inside the aquifer (Dupuit-Forchheimer assumption) is even less accurate due to the reduced hydraulic conductivity of the center layer. Including these resistances to vertical flow as done for the case of Figure 7 and Figure 8, will reduce the error some, as seen before.

Multiple Aquifers

The contrast in hydraulic conductivity in the previous problem is a factor 5, which is border line for accepting the Dupuit-Forchheimer approximation. Consequently, the discrepancy in the flow between the short and the long model at 45 meters to the right of the ditch was as "high" as 3%. If the center layer would have a hydraulic conductivity that is several orders lower than that in the upper and lower zone, the stratified aquifer becomes a system of two aquifers separated by an aquitard. For that case the calculation of a characteristic leakage length for the entire system, using (17), becomes meaningless.

The exact solution for the groundwater flow in each of the aquifers of a multi-aquifer system is given in the literature, for instance, see Bruggeman (1999) page 645 Example 7 which deals with one dimensional flow in two aquifers underneath a lake or wetland (the expressions are omitted here as they are quite complex). In example 7 by Bruggeman the boundary condition at one end of the aquifers is a given drop in the head h, in each aquifer, relative to the head in the wetland. For our case the head difference h will be different in each aquifer. As may be seen from the exact solution the heads and thus the specific discharges in each aquifer involves not only the difference between the head in that aquifer and the head in the wetland, but also the differences between the heads in the adjacent aquifers and the wetland head. This precludes the use of the exact solution to define a representative resistance for a line-sink string as was done in the previous cases. The problem is that the heads in the adjacent aquifers, at the location of the line-sink strings, are not known in advance and, as mentioned, differ from the head in the aquifer in question.

Approximate procedure

We will approximate the remote boundary conditions in a manner similar to the approach outlined in the section "Refinement of the resistance on boundary line-sinks". We will treat the flow in each of the aquifers as that occurring in a single aquifer separated from the wetland by an aquitard of resistance c. For the upper aquifer that resistance is simply the resistance of the wetland bottom. For the lower aquifer, however, the resistance is the sum of the resistances to vertical flow through the wetland bottom, upper aquifer, and aquitard between the aquifers.

We will calculate a characteristic leakage length λ for each of the two aquifers. For the upper aquifer we get with (4):

$$\lambda = \sqrt{kHc_w} = \sqrt{100 * 5 * 1} = 22.3607 \; meters \tag{21}$$

For the lower aquifer we must include the resistance to vertical flow through the upper aquifer and the aquitard, hence $c_w = 1 + 5/100 + 10/1 = 11.05 \; days$. The characteristic leakage length for the lower aquifer than becomes:

$$\lambda = \sqrt{kHc_w} = \sqrt{100 * 5 * 11.05} = 74.3303 \; meters \tag{22}$$

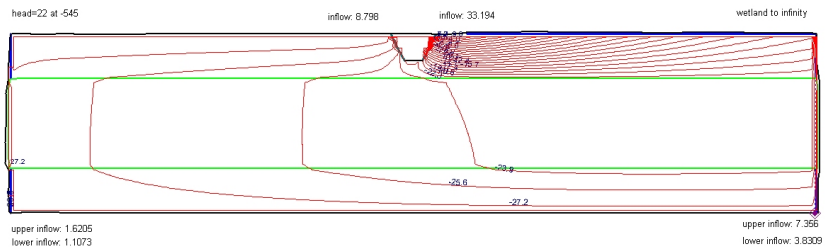

Figure 11: Streamlines (stream function contours) near ditch. Line-sinks along the right-hand and left-hand boundaries have different resistances depending on the aquifer layer they are in. Center layer has 100 times lower conductivity than adjacent layers. Project file: *example4.gfl*

We will give the line-sinks on the right-hand boundary of the two aquifers each a single resistance, calculated from (7). This yields for the upper aquifer a resistance:

$$c = \frac{\lambda}{k} = \frac{22.3607}{100} = 0.223607 \; days \tag{23}$$

While for the lower aquifer:

$$c = \frac{\lambda}{k} = \frac{74.3303}{100} = 0.743303 \; days \tag{24}$$

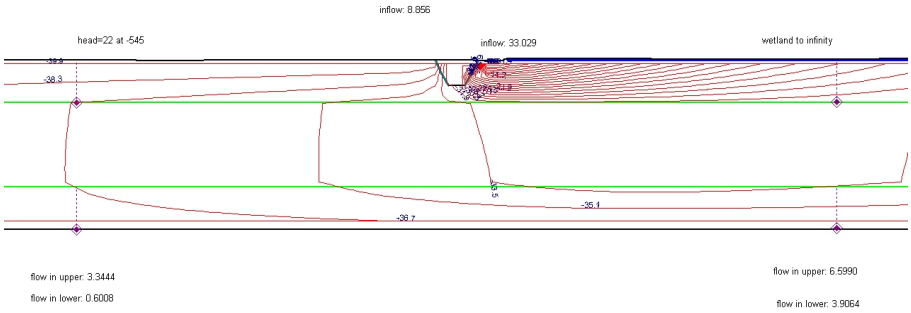

Figure 12: Streamlines (stream function contours) in extended model domain, but zoomed in to about the same domain as shown in Figure 7. Line-sinks along the right-hand and left-hand boundaries have different resistances depending on the aquifer layer they are in. Center layer has 100 times lower conductivity than adjacent layers. Project file: *example4long.gfl*

Comparing the solution for the short model, see Figure 11, with that for the long model, see Figure 12, we find the following. While visually the solutions seem quite similar (similar streamline patterns) there are some discrepancies in the flow rates. The total flow into the ditch is 0.25% more in the short model than in the long model, which is an insignificant difference. However, the flow rates in the upper and lower aquifer of the short model, at 45 meters to the right of the center of the ditch (underneath the wetland), are respectively 11% lower and 1.7% higher than in the long model. On the left-hand side in the short model, at 45 meters from the center of the ditch, the flows in the upper and lower aquifer are 50% lower and 80% higher than in the long model. However, these are discrepancies in rather small flow rates, see Figure 11 and Figure 12, that have only a limited effect on the overall flow pattern; compare the streamlines in both figures. On the other hand, the errors at the boundaries are much larger than in the previous examples, demonstrating that, for cases of multi-aquifer flow, care must be taken when truncating long models by use of line-sink strings with a representative resistance.

When applying this procedure to a specific case of multi-aquifer flow it is recommended that, whenever possible, the boundary underneath the wetland is placed at a distance of several times the largest λ-value from the area of interest (center of the ditch in the examples). When doing so, the vast majority of the leakage from the wetland occurs inside the model domain. Consequently, any errors in flow from the aquifer (underneath the wetland) outside the model domain concerns only a small percentage of the overall water balance. The model domains in Figure 11 and Figure 12 do not extend very far underneath the wetland. The largest λ-value is 74 meters, while in the short model the line-sinks were placed at 45 meters and in the long model at 134 meters from the center of the ditch.

Summary

Cross-sectional models may be much longer than they are high. This awkward aspect ratio makes for difficult model setup and can lead to solution instabilities or inaccuracies. The model domain may be truncated at some distance from the area of interest, provided that the flow from the remote aquifer zones that are not included in the model is properly accounted for in the local (truncated) model domain. Remote head specified boundaries can be simulated by means of a "general head boundary," which is a Cauchy type boundary that includes the resistance to horizontal flow in the aquifer zone that is outside the model domain. The resistance at the model boundary (e.g. head specified line-sinks with resistance) is $c = L/k$ [days] where L is the distance between the model boundary and the remote Dirichlet boundary and where k is the aquifer hydraulic conductivity.

It appears that the flow resulting from an aquifer underneath a wetland or lake with a bottom resistance c_w can also be included by use of a Cauchy type boundary. For this case the resistance $c = \lambda/k$. The parameter λ is the

characteristic leakage length defined as $\lambda = \sqrt{kHc_w}$, where H is the aquifer thickness. These surrogate boundaries are approximate in that the Dupuit-Forchheimer approximation is adopted for flow in the aquifer zones outside the model domain, hence resistance to vertical flow is ignored in those remote aquifer zones. While usually valid in the remote confined aquifer, there is some vertical flow in the remote aquifer underneath the wetland, so that the Cauchy boundary will only approximate the flow from that aquifer zone.

The approach to truncating a sectional model in lateral direction can be extended to include aquifer stratification and even applied to cases of multi-aquifer flow. However, the approximation of the flow from the remote wetlands is less accurate for the case of stratified flow and worse yet for multi-aquifer flow. However, if it is practical to make the model domain include a section of the wetlands with a length several times the characteristic leakage length λ, the approximation will have little or no effect on the model accuracy. This is due to fact that in the case of a single aquifer underneath a wetland, 95% of the leakage from the wetland occurs within a distance of 3λ from a boundary condition, say the ditch in Figure 1 (Hunt et al. 2003).

References

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