Implementation of Horizontal Barriers in GFLOW

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Horizontal Barriers in GFLOW

The modeling of "horizontal barriers" in GFLOW or "slurry walls" as they are called in the GFLOW Solver "GFLOW1.exe" is based on the concept of using the stream function for flow governed by Poisson's equation (Haitjema and Kelson, 1996). The crux of the matter is that in order to calculate the flow across a line segment (straight barrier section) the difference in stream function values, which equals flow due to all functions satisfying Laplace's equation, can be augmented with the integral of the (few) functions that satisfy Poisson's equation to arrive at the total flow across the line segment. To define a no-flow boundary, or a boundary with a given resistance to flow (slurry wall), appropriate boundary conditions may be imposed at collocation points along the slurry wall. However, such an approach only ensures that the condition is met at a number of discrete (collocation) points, while in between the no-flow boundary may leak or the resistance boundary may have a larger or smaller resistance. In GFLOW the flow across a segment of a barrier is considered when setting boundary conditions for a barrier. These integrated boundary conditions are more robust than the collocation point method, see Haitjema and Kelson (1996).

The GFLOW solver (GFLOW1.exe) refers to all barriers as "slurry walls," which may be no-flow boundaries, fully penetrating resistance boundaries, or partially penetrating resistance boundaries. Instead of a resistance to flow, the inverse, the *conductance*, is calculated for each of these cases and enforced along boundary segments as detailed below.

Conductance for a slurry wall

With reference to Figure 1 the following parameters are defined: The width and conductivity of the slurry wall are w $[m]$ and k_c $[m/day]$, respectively. The head to the left and the right of the wall are ϕ_1 $[m]$ and ϕ_2 $[m]$, respectively, both measured with respect to the aquifer base. The average saturated thickness of the "aquifer" inside the slurry wall is $\bar{h}$ $[m]$, which for the case of unconfined

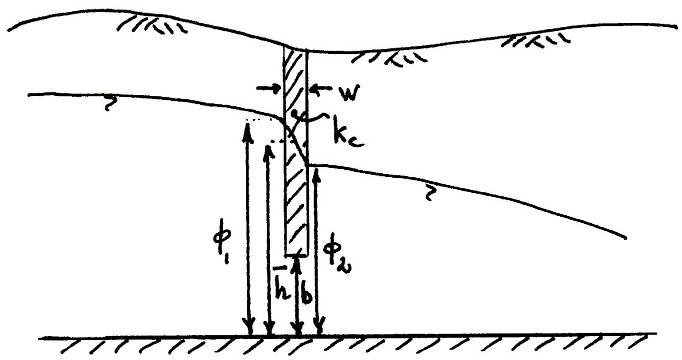

Figure 1: Partially penetrating slurry wall.

flow is, see Figure 1:

$$\bar{h} = \frac{\phi_1 + \phi_2}{2} \tag{1}$$

or for the case of confined flow (not depicted):

$$\bar{h} = H \tag{2}$$

where H $[m]$ is the (constant) aquifer thickness. The gap between the aquifer bottom and the slurry wall is b $[m]$. The hydraulic conductivity in the aquifer outside the slurry wall is k $[m/day]$.

For the general case depicted in Figure 1 the conductance C $[1/day]$ is:

$$C = \frac{(\bar{h} - b)k_c + bk}{\bar{h}w} \tag{3}$$

No gap

For the case that there is no gap, hence $b = 0$, the discharge rate Q_n $[m^2/day]$ through the slurry wall follows from Darcy's law (Haitjema, 1995):

$$Q_n = \frac{\overset{c}{\Phi}_1 - \overset{c}{\Phi}_2}{w} \tag{4}$$

where $\overset{c}{\Phi}$ $[m^3/day]$ is the discharge potential inside the slurry wall, defined as:

$$\overset{c}{\Phi} = \tfrac{1}{2} k_c \phi^2 \tag{5}$$

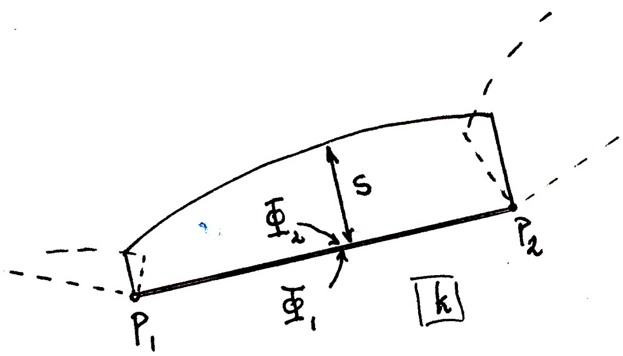

Figure 2: Jump in the potential across a line-doublet from point p_1 to point p_2.

Equation (4) with (5) becomes:

$$Q_n = \frac{\frac{1}{2}k_c(\phi_1^2 - \phi_2^2)}{w} \tag{6}$$

The slurry wall will be represented by a string of straight line-doublets, one of which is depicted in Figure 2. The line-doublet strength s $[m^3/day]$ equals the jump in the aquifer potentials across the wall:

$$s = \Phi_2^2 - \Phi_1^2 \tag{7}$$

which written in terms of the heads ϕ_1 and ϕ_2 is:

$$s = \tfrac{1}{2}k(\phi_2^2 - \phi_1^2) \tag{8}$$

Note, the jump is the potential to the left of the line-doublet minus the potential to the right of the line-doublet when travelling from the starting point p_1 to the end point p_2. Combining (4) and (8) yields:

$$Q_n = -\frac{k_c s}{k w} \tag{9}$$

Introducing the conductance C $[1/day]$ as:

$$C = \frac{k_c}{w} \tag{10}$$

yields for the discharge Q_n:

$$\boxed{Q_n = -\frac{C}{k}s \qquad (11)}$$

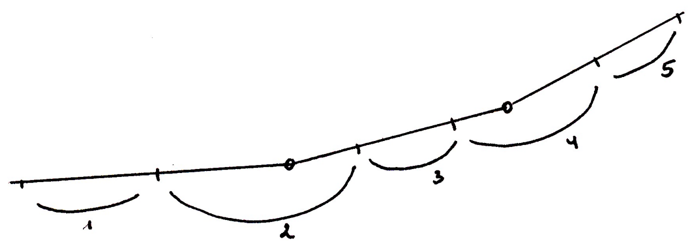

Figure 3: Open ended slurry wall with "integration segments" for implementing boundary conditions.

With gap

The analysis presented above expresses the discharge rate through the barrier in terms of a conductance C of the barrier and the aquifer conductivity k and line-doublet strength s. The same equation (11) applies for a slurry wall with a gap b between the aquifer bottom and the slurry wall if the definition of C, as given by (10), is replaced by (3).

Defining Boundary Conditions

In GFLOW1 there are two types of slurry walls:

- Open ended slurry walls.
- Closed slurry walls.

These two slurry walls have their own characteristics when including them in the solution procedure. A detailed analysis for setting the boundary conditions along these slurry walls is given in Haitjema and Kelson, 1996, sections 8 and 9. A brief conceptual description and some implementation details for GFLOW1 are presented below.

Open ended slurry walls

In Figure 3 an open ended slurry wall is depicted with three line-doublets. Each line-doublet has a second order (parabolic) strength distribution, which is defined by three strength parameters per line-doublet, one for each endpoint, which together define the linear component of the strength distribution, and a parabolic component defined at the center of each line-doublet, see Haitjema

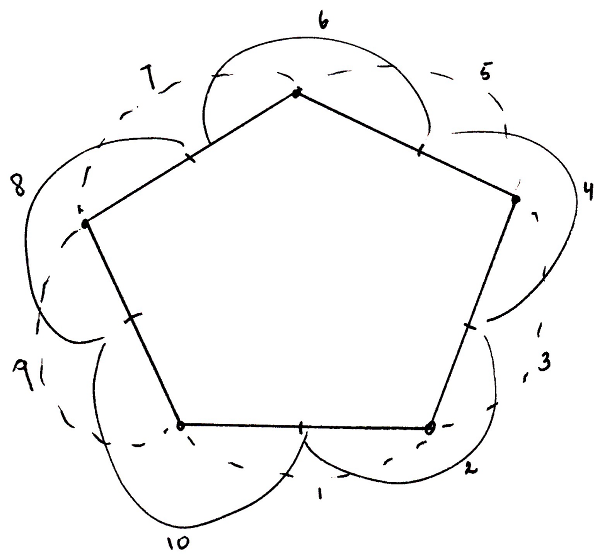

Figure 4: Closed slurry wall with "integration segments" for implementing boundary conditions.

and Strack, 1981b. As a result, for the three line-doublets depicted in Figure 3, there are seven (7) strength parameters that need to be selected such as to satisfy the specified boundary conditions along the slurry wall. Two of these, however, are known in advance: the strengths at the endpoints are zero. There cannot be a potential jump at the extremeties of the slurry wall since that would imply two different heads (thus two different pore pressures) in one point (end point), which is physically impossible. This leaves 5 unknown strength parameters to be calculated during the solution procedure in GFLOW1. As explained in Haitjema and Kelson (1996), the condition (11) is integrated along a segment of the slurry wall to arrive at a boundary condition. For instance, in the event of a no-flow boundary the flow Q_n, integrated along a segment, and generated by all analytic elements, including those of the no-flow boundary itself, must be zero. Since we must select 5 strength parameters we need 5 integration segments. In GFLOW1 these are selected from a point very close, but not at, the endpoints to the center of the ending line-doublets (2 segments) and from the center of each line-doublet to the center of the adjacent line-doublet, see Figure 3.

Closed slurry walls

Unlike for open ended slurry walls, there are no a priory known line-doublet strengths parameters for closed slurry walls. In Figure 4 a closed slurry wall of

5 line-doublets is depicted. This requires the calculation of 10 strength parameters. In GFLOW1 two different integration paths are selected: one across each vertex (line-doublet end point) and one at the center of each line-doublet. The integration paths may be selected to be contiguous or the may *overlap*.

In the event that the closed slurry wall is a no-flow boundary, two independent groundwater flow domains are created: the *inside* and the *outside* domain. In the GFLOW or WhAEM GUI a reference point with a reference head is selected automatically outside the model domain, ensuring at least one head specified condition in the outside domain. However, it is up to the user to ensure that the closed no-flow boundary is given an *inside reference head*, for instance by some head specified analytic element (well or line-sink). The GFLOW1 solver will abort when it does not find at least one head specified boundary condition inside a closed no-flow boundary.

Pathline Tracing through Slurry Walls

For the case of Figure 1 there is flow through and underneath the slurry wall. Denoting the flow through the wall as Q_w we may write:

$$\frac{Q_w}{Q_n} = \frac{(\bar{h} - b)k_c}{(\bar{h} - b)k_c + bk} \tag{12}$$

which with (3) becomes:

$$\frac{Q_w}{Q_n} = \frac{(\bar{h} - b)k_c}{Cw\bar{h}} \tag{13}$$

With (11) the discharge through the wall can be written as:

$$Q_w = \frac{(\bar{h} - b)k_c}{Cw\bar{h}} \left(-\frac{C}{k} \right) s = -\frac{(\bar{h} - b)k_c}{w\bar{h}k} s \tag{14}$$

The average groundwater flow velocity in the slurry wall, v_w $[m/day]$, is obtained by dividing the discharge by the saturated height of the wall $(\bar{h} - b)$ and its effective porosity n_w $[-]$:

$$v_w = \frac{Q_w}{(\bar{h} - b)n_w} \tag{15}$$

which becomes with (14):

$$\boxed{v_w = -\frac{k_c s}{n_w k w \bar{h}}} \tag{16}$$

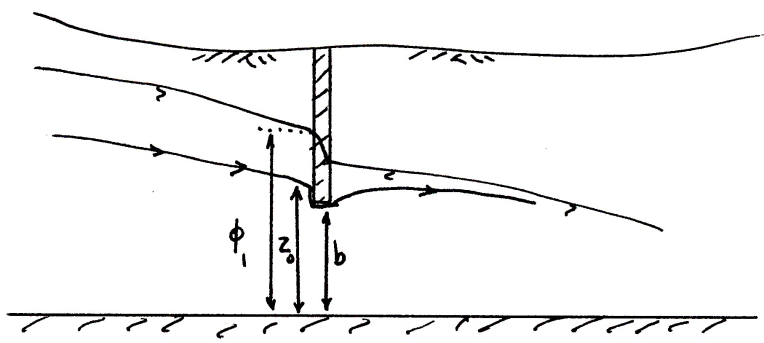

Figure 5: Dividing pathline at elevation z_0 between water that travels through and underneath the slurry wall.

Residence time

The residence time of a water particle that travels through the slurry wall, t_w $[days]$ is:

$$t_w = \frac{w}{|v_w|} \tag{17}$$

which with (16) becomes

$$t_w = \frac{n_w k w^2 \bar{h}}{k_c |s|} \tag{18}$$

Elevation of path line through wall bottom

In order to assess whether or not a pathline travels through or underneath a slurry wall the dividing pathline elevation z_0 $[m]$ must be calculated, see Figure 5. The dividing pathline elevation z_0, which is the elevation of the streamline that will just go underneath the slurry wall, rather than through the slurry wall, follows from:

$$\frac{z_0}{\phi_1} = \frac{Q_n - Q_w}{Q_n} \tag{19}$$

so that with reference to (12) we write:

$$z_0 = \frac{bk\phi_1}{(\bar{h} - b)k_c + bk} \tag{20}$$

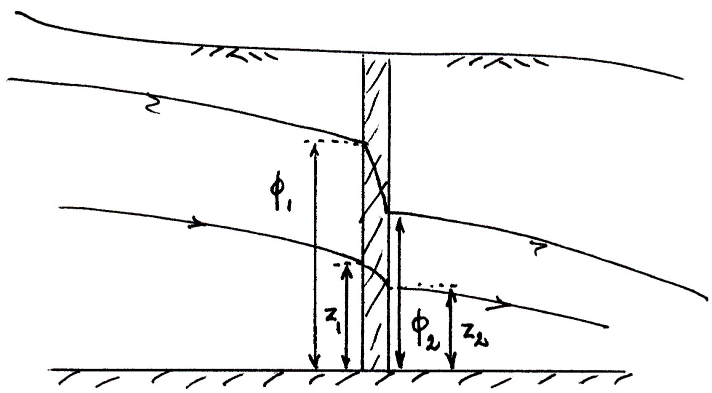

Figure 6: Jump $(z_2 - z_1)$ in pathline elevation across the slurry wall.

Consequently, if the pathline arrives at the slurry wall at an elevation larger than z_0 the water particle must travel through the wall and a residence time t_w, defined by (20) must be added to the current time. Note, that for confined flow conditions the saturated thickness ϕ_1 must be replaced by the aquifer thickness H.

Jump in pathline elevation

Under unconfined flow conditions the water table elevation will jump across the wall and so will a pathline elevation that crosses that wall (whether it actually goes through the wall or travels underneath it), see Figure 6. The jump in the pathline elevation from z_1 to z_2 across the slurry wall follows from:

$$\frac{z_1}{z_2} = \frac{\phi_1}{\phi_2} \tag{21}$$

so that the "new" pathline elevation z_2 can be expressed in terms of the "old" elevation z_1 as:

$$z_2 = \frac{\phi_2}{\phi_1} z_1 \tag{22}$$

Equation (8) relates the heads on either side of the wall to the line-doublet strength:

$$s = k(\phi_2 - \phi_1)\frac{\phi_1 + \phi_2}{2} \tag{23}$$

which becomes with (1):

$$s = k(\phi_2 - \phi_1)\bar{h} \tag{24}$$

so that

$$\phi_2 = \frac{s}{k\bar{h}} + \phi_1 \tag{25}$$

With (25) we write for (22):

$$\boxed{z_2 = z_1 \left(\frac{s}{k\bar{h}\phi_1} + 1 \right) \qquad (26)}$$

References

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